





All multipartite entangled states of qubits can be self-tested

Ivan Šupić

QIP, Riga 29/01/2026

Collaborators



Maria Balanzo-Juando
ULB
Brussels



Andrea Coladangelo
University of Washington
Seattle



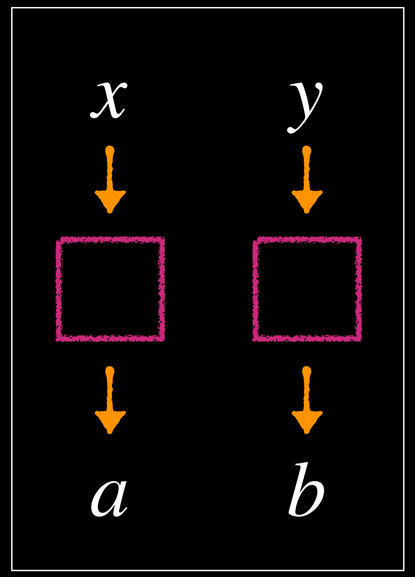
Remigiusz Augusiak
Center for Theoretical Physics
Warsaw



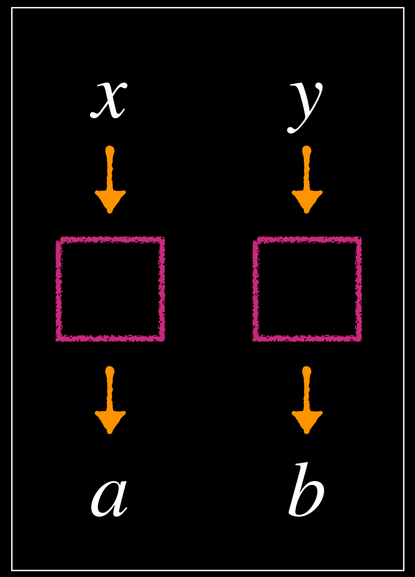
Antonio Acin
ICFO
Barcelona

All pure multipartite qubit states can be self-tested up to complex conjugation
M. Balazo-Juando, A. Coladangelo, R. Augusiak, A. Acin, IŠ
arXiv: 2412.13266

Self-testing



Self-testing



$$\{p(a,b\,|\,x,y)\}_{a,b}^{x,y}$$

$x = 0, y = 0$

	0	1
0	0,43	0,07
1	0,07	0,43

$x = 0, y = 1$

	0	1
0	0,43	0,07
1	0,07	0,43

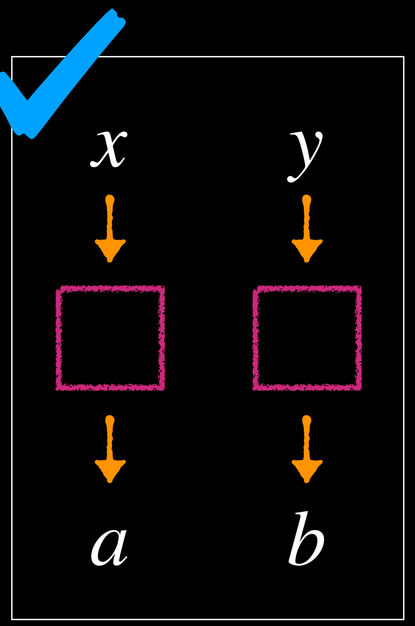
$x = 1, y = 0$

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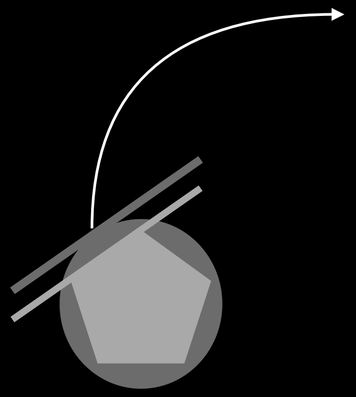
$x = 1, y = 0$

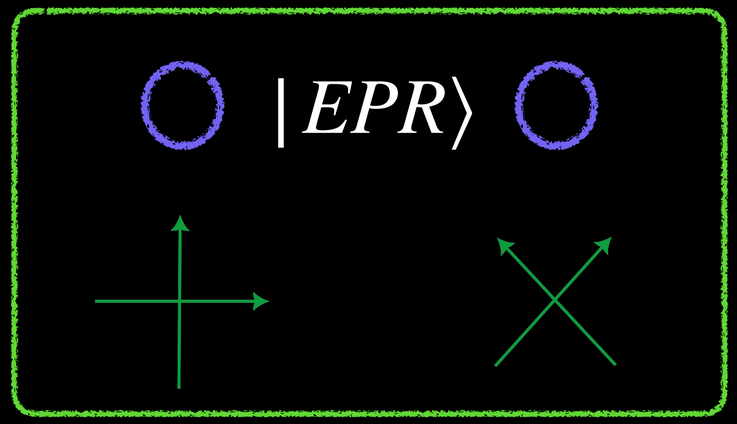
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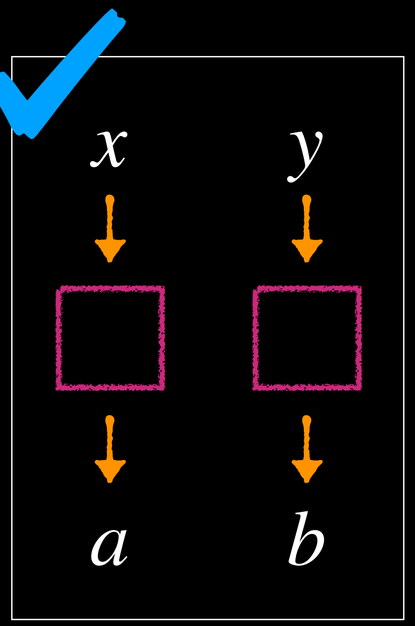
$$p(a,b\,|\,x,y) = \mathrm{Tr}\left[M_x^a \otimes N_y^b \varrho\right]$$





Summers, Werner 1987

Self-testing



$$\{p(a,b\,|\,x,y)\}_{a,b}^{x,y}$$

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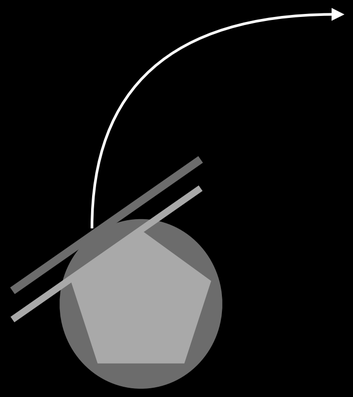
$x=1, y=0$

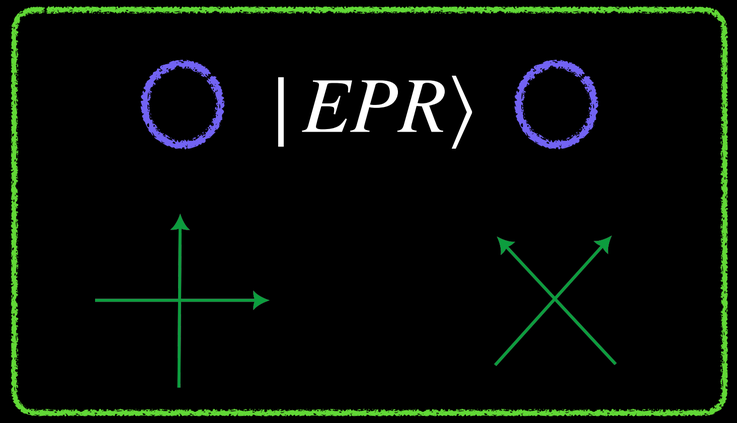
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$x=1, y=1$

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1	0,43	0,07

$$p(a,b\,|\,x,y) = \mathrm{Tr}\left[M_x^a \otimes N_y^b \varrho\right]$$





Summers, Werner 1987

Device-independent certification

Bancal, Redeker, Sekatski, Rosenfeld, Sangouard 2021

Randomness amplification

Pironio et al 2010

Complexity

Ji, Natarajan, Vidick, Wright, Yuen 2020

Cryptography

Wooltorton, Brown, Colbeck 2024

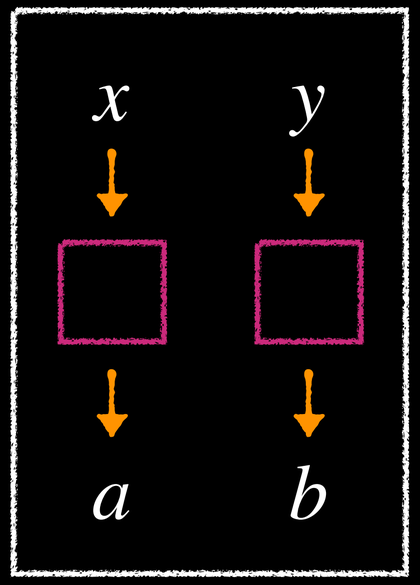
Foundations

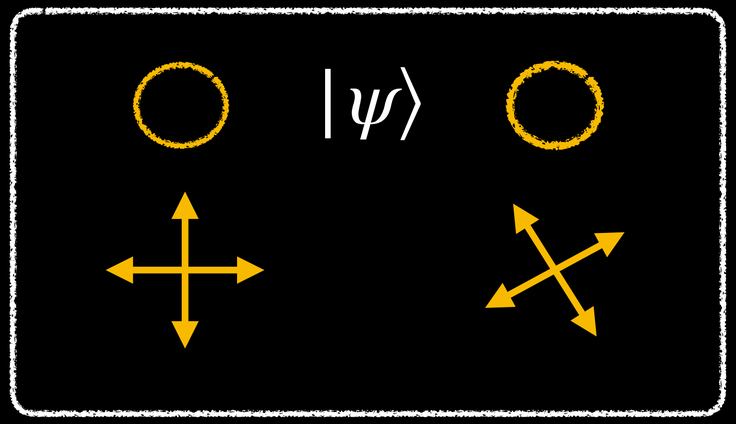
Barizien, Bancal 2025

Delegated computing

Coladangelo, Grilo, Jeffery, Vidick 2024

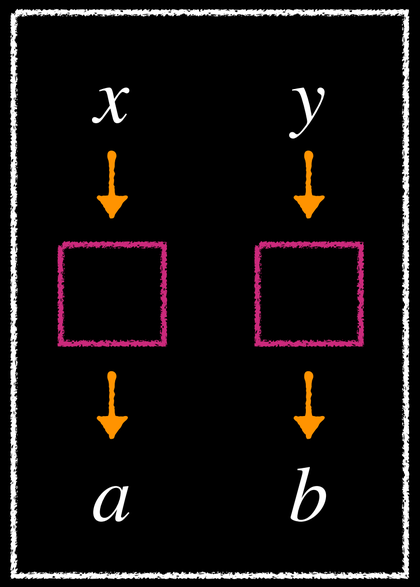
Self-testing

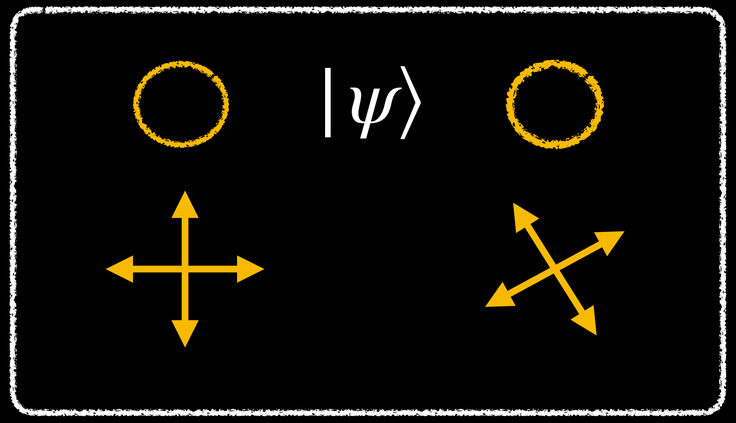


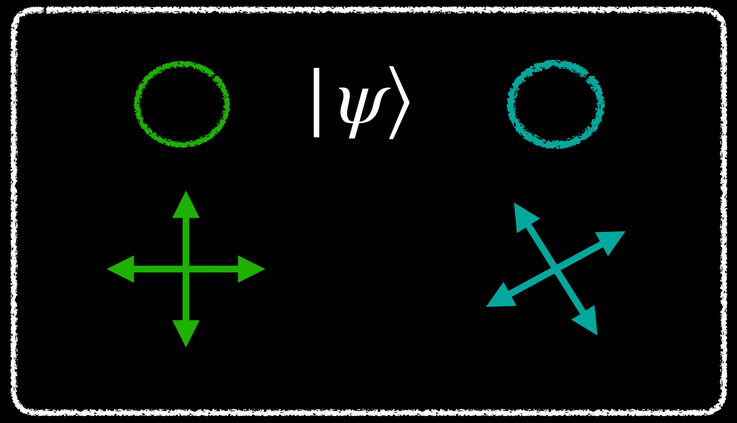


$$\left[|\psi\rangle, \quad M_x^a, \quad N_y^b \right]$$

Self-testing



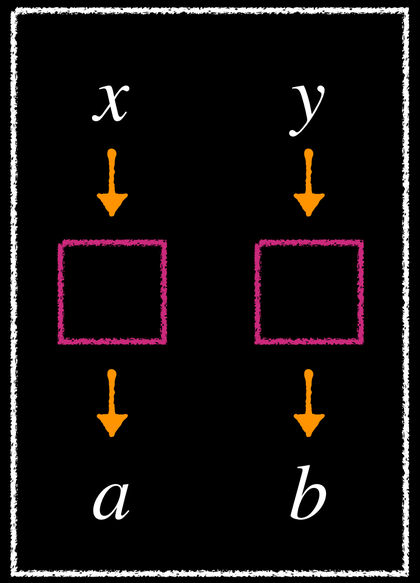


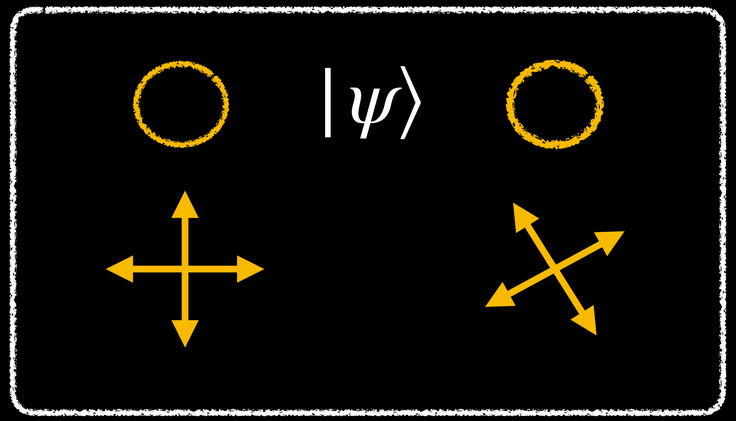


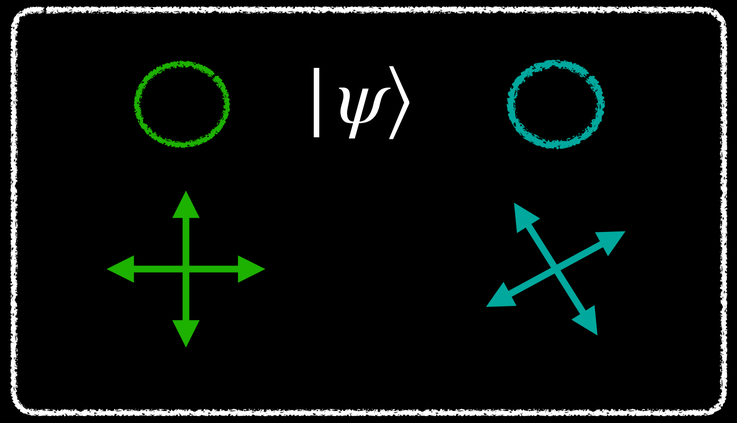
$$\left[|\psi\rangle,\quad M_x^a,\quad N_y^b\right]$$

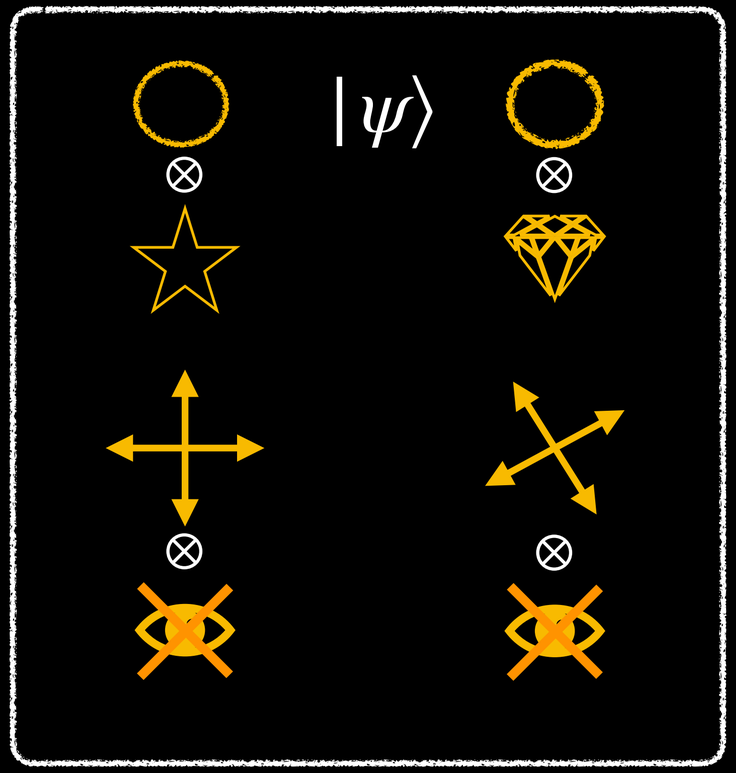
$$\left[U\otimes V|\psi\rangle,\quad UM_x^aU^\dagger,\quad VN_y^bV^\dagger\right]$$

Self-testing







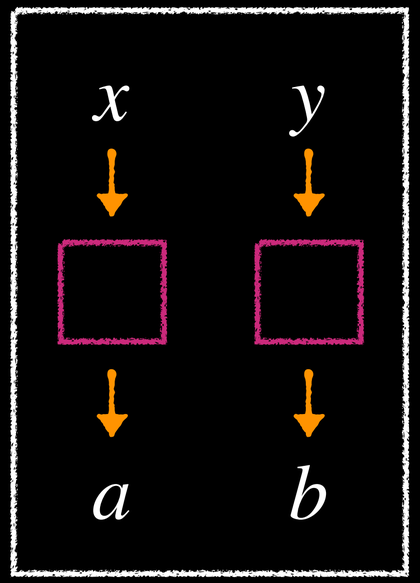


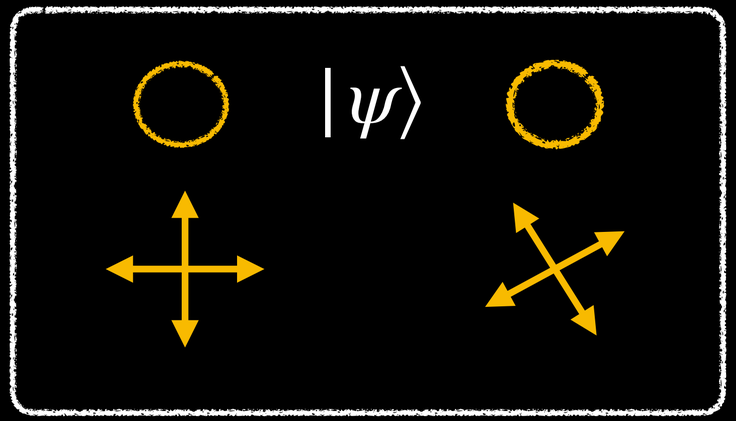
$$\Big[\,|\psi\rangle,\quad M_x^a,\quad N_y^b\,\Big]$$

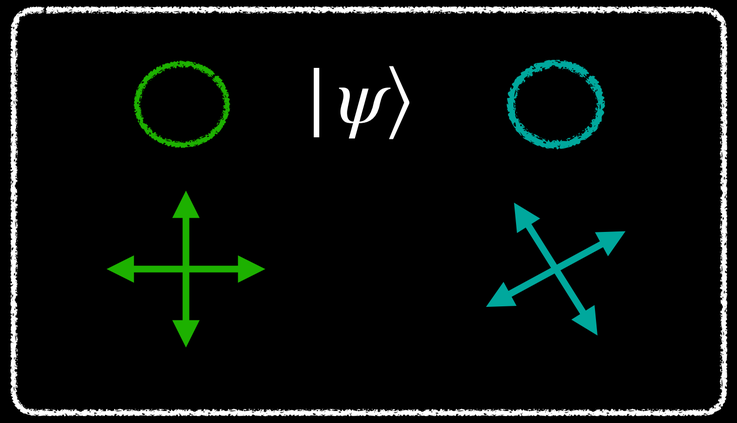
$$\Big[\,U\otimes V|\psi\rangle,\quad UM_x^aU^\dagger,\quad VN_y^bV^\dagger\,\Big]$$

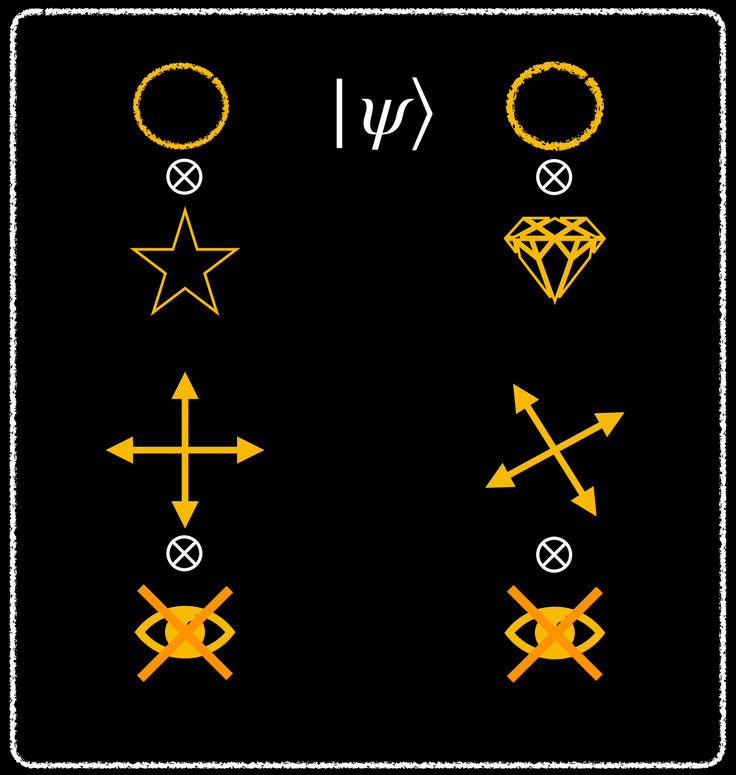
$$\Big[\,|\psi\rangle\otimes|\diamond\rangle,\quad M_x^a\otimes 1,\quad N_y^b\otimes 1\,\Big]$$

Self-testing









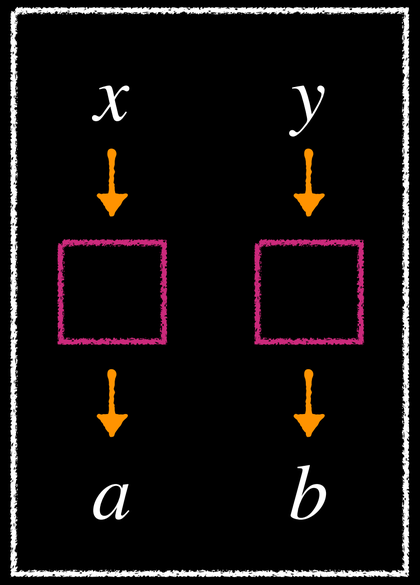
$$\Big[\,|\psi\rangle,\quad M_x^a,\quad N_y^b\,\Big]$$

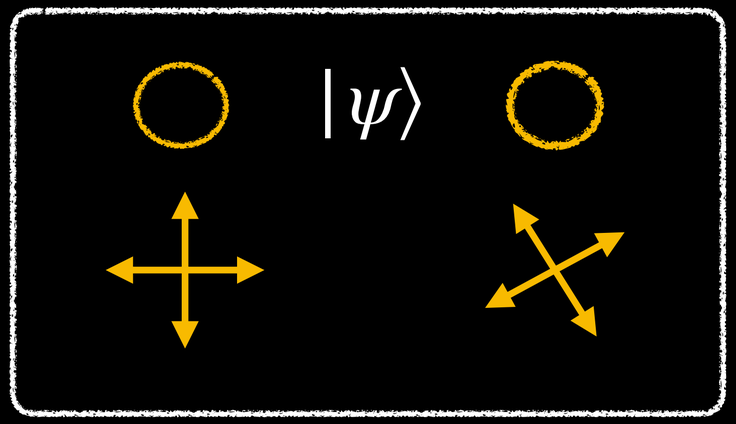
$$\Big[\,U\otimes V|\psi\rangle,\quad UM_x^aU^\dagger,\quad VN_y^bV^\dagger\,\Big]$$

$$\Big[\,|\psi\rangle\otimes|\diamond\rangle,\quad M_x^a\otimes 1,\quad N_y^b\otimes 1\,\Big]$$

Local isometries

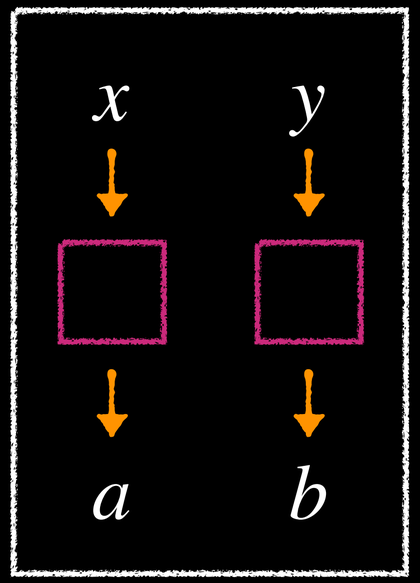
Self-testing

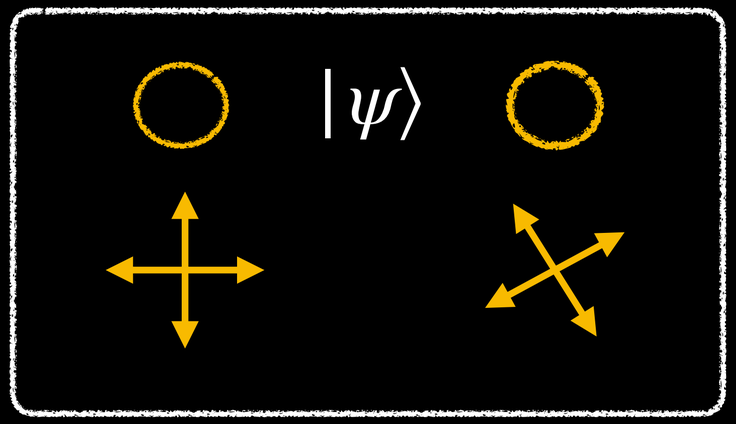




$$\left[|\psi\rangle, \quad M_x^a, \quad N_y^b\right]$$

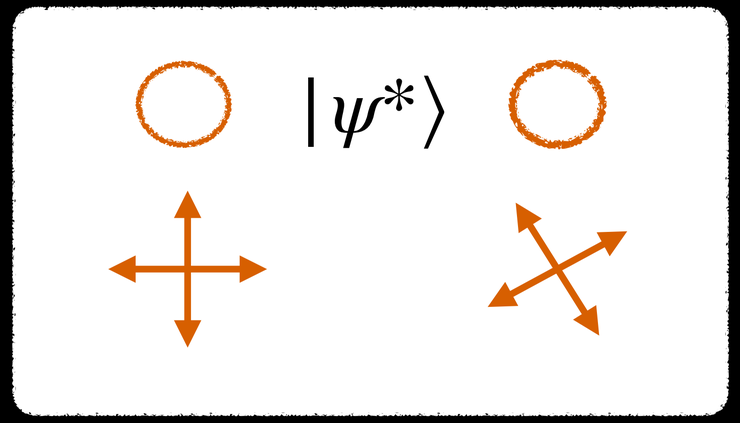
Self-testing



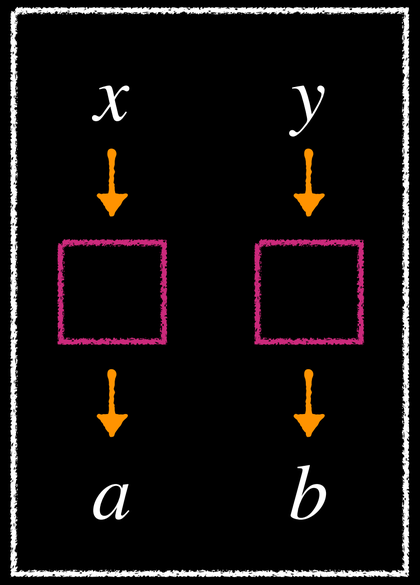


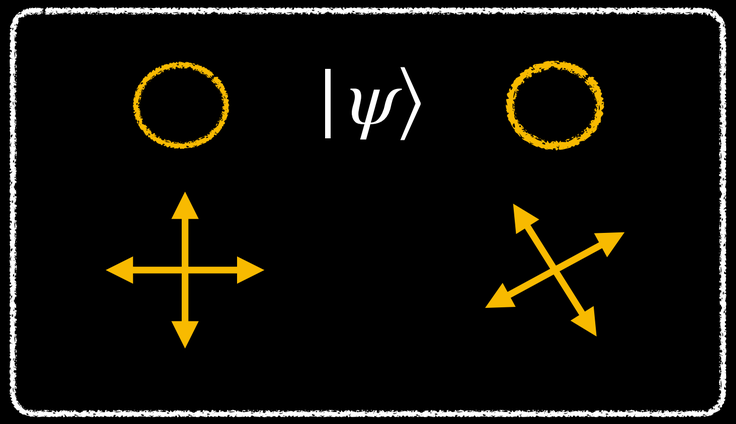
$$\left[|\psi\rangle, \quad M_x^a, \quad N_y^b\right]$$

$$\left[|\psi^*\rangle, \quad M_x^{a^*}, \quad N_y^{b^*}\right]$$

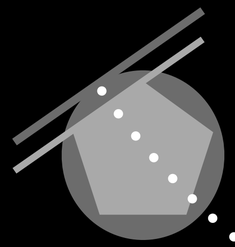


Self-testing

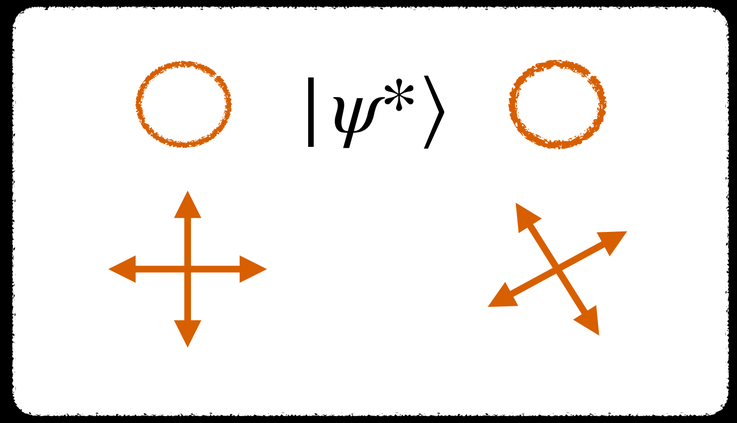




$$\left[|\psi\rangle, \quad M_x^a, \quad N_y^b\right]$$



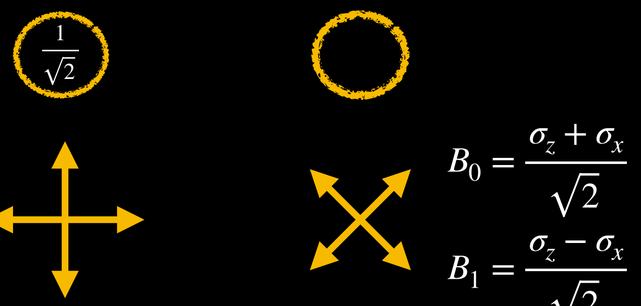
$$\left[|\psi^*\rangle, \quad M_x^{a^*}, \quad N_y^{b^*}\right]$$



$$\Phi_A \otimes \Phi_B |\psi'\rangle = p\,|\psi\rangle \otimes |aux_0\rangle + \sqrt{1-p^2}\,|\psi^*\rangle \otimes |aux_1\rangle$$

Bipartite states

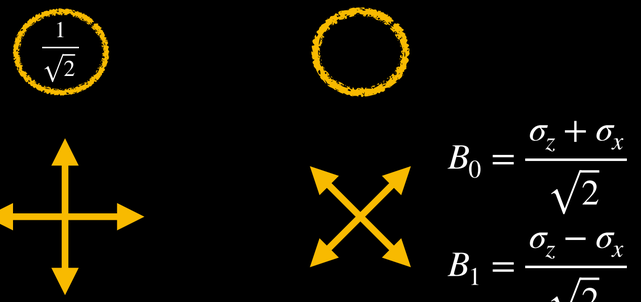
Maximally entangled pair



Summers, Werner 1987

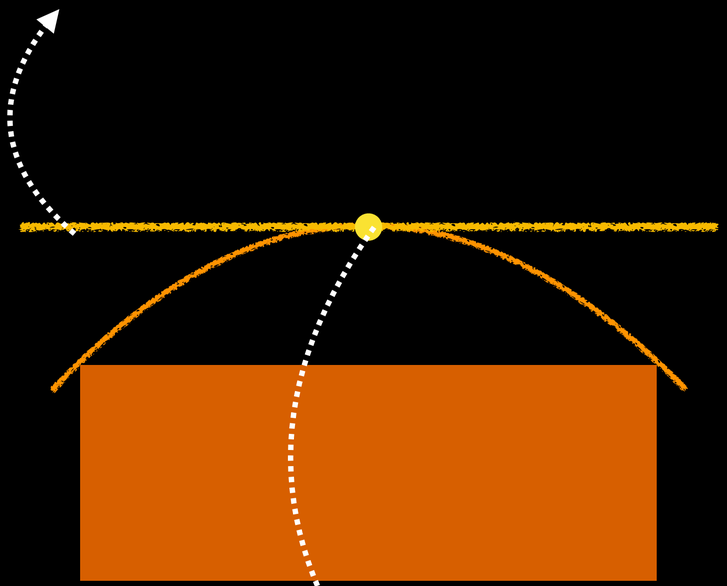
Bipartite states

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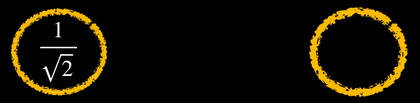
$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle = 2\sqrt{2}$$



$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right)$$

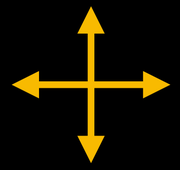
Bipartite states

Maximally entangled pair



$A_0 = \sigma_z$

$A_1 = \sigma_x$





$B_0 = \frac{\sigma_z + \sigma_x}{\sqrt{2}}$

$B_1 = \frac{\sigma_z - \sigma_x}{\sqrt{2}}$

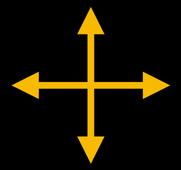
Summers, Werner 1987

Partially entangled pair



$A_0 = \sigma_z$

$A_1 = \sigma_x$





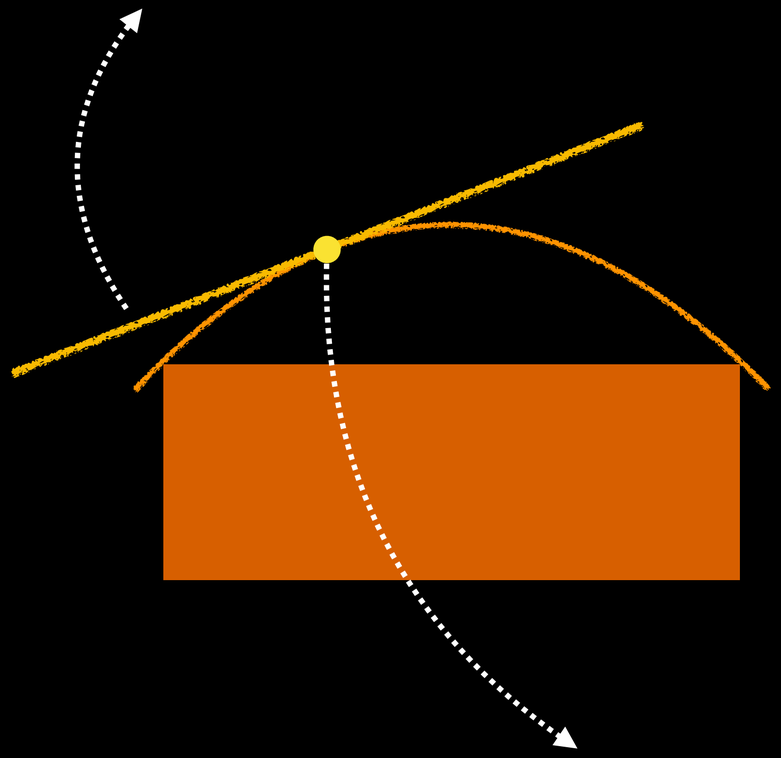
$B_0 = \cos\mu\sigma_z + \sin\mu\sigma_x$

$B_1 = \cos\mu\sigma_z - \sin\mu\sigma_x$

Bamps, Pironio 2015

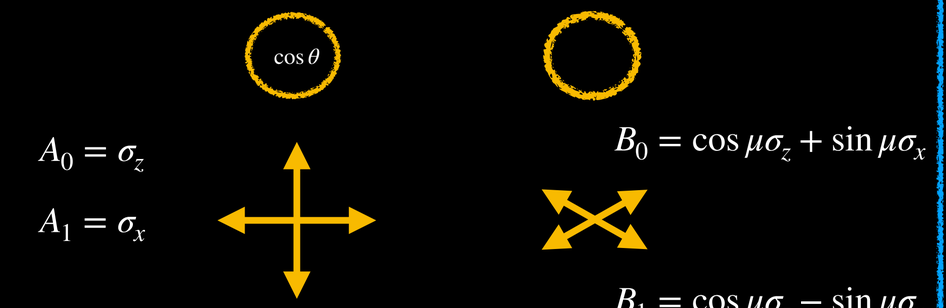
Bipartite states

$$\alpha\langle A_0\rangle + \langle A_0B_0\rangle + \langle A_0B_1\rangle + \langle A_1B_0\rangle - \langle A_1B_1\rangle = \sqrt{8+\alpha^2}$$



$$|\psi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$$

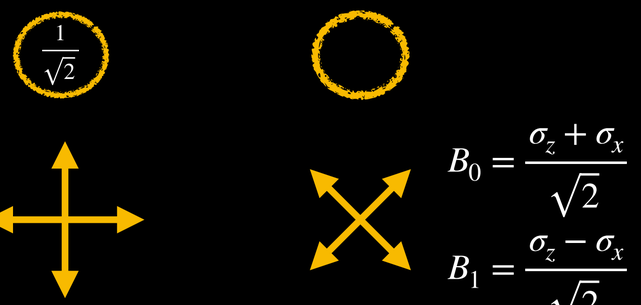
Partially entangled pair



Bamps, Pironio 2015

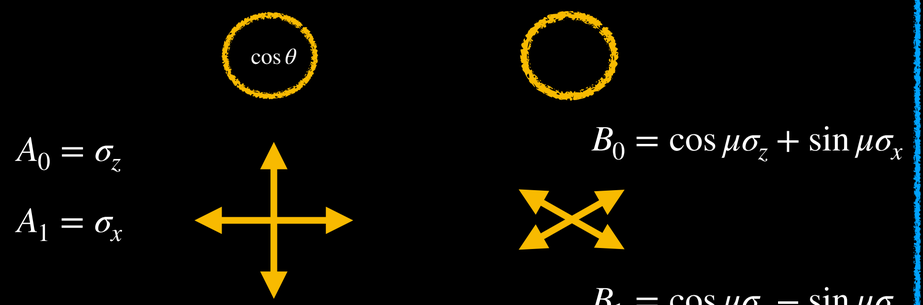
Bipartite states

Maximally entangled pair



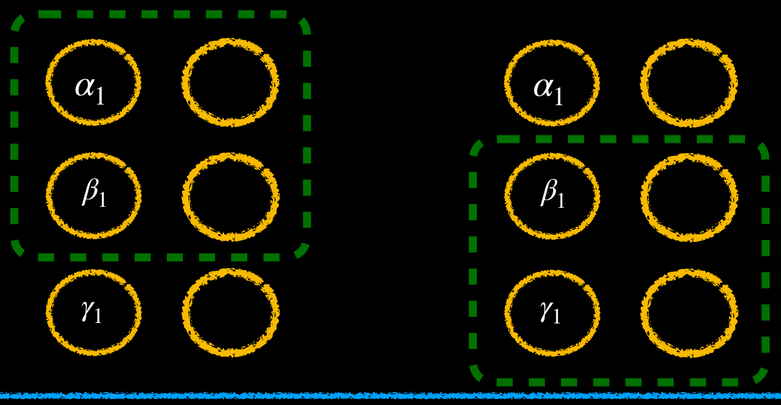
Summers, Werner 1987

Partially entangled pair



Bamps, Pironio 2015

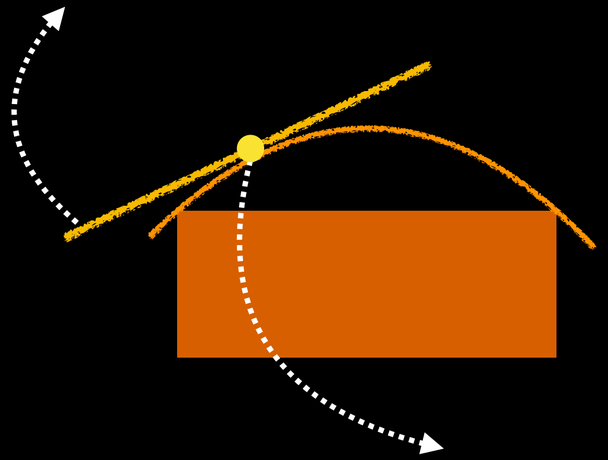
Arbitrary bipartite pure state



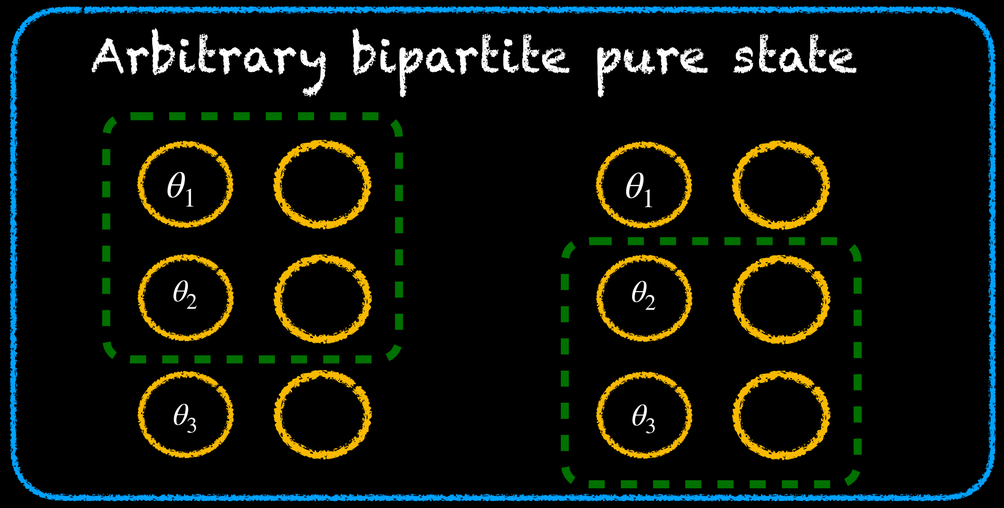
Coladangelo, Goh, Scarani 2017

Bipartite states

$$\alpha_1\langle A_0\rangle + \langle A_0B_0\rangle + \langle A_0B_1\rangle + \langle A_1B_0\rangle - \langle A_1B_1\rangle = \sqrt{8+\alpha_1^2}$$

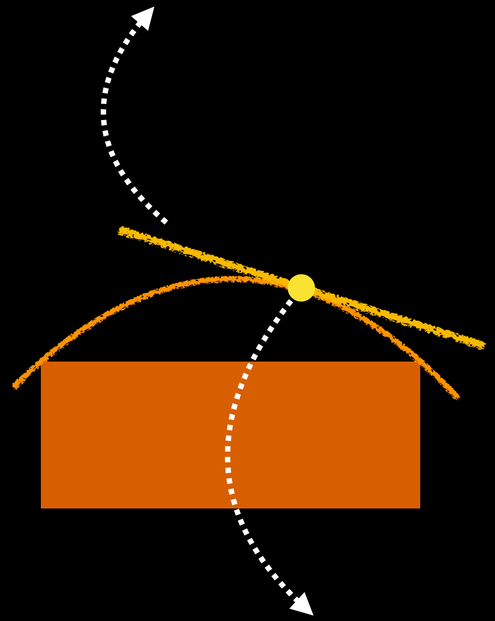


$$|\psi_1\rangle = \cos\theta_1\,|00\rangle + \sin\theta_1\,|11\rangle$$



Coladangelo, Goh, Scarani 2017

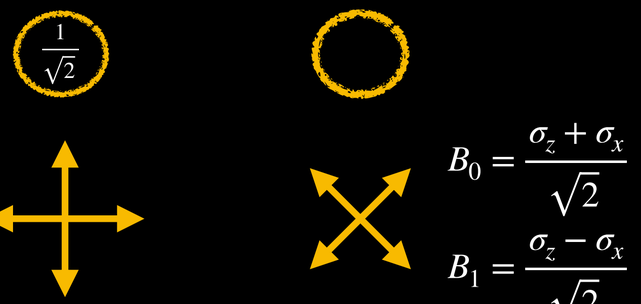
$$\alpha_2\langle A_0\rangle + \langle A_0B_0\rangle + \langle A_0B_1\rangle + \langle A_1B_0\rangle - \langle A_1B_1\rangle = \sqrt{8+\alpha_2^2}$$



$$|\psi_2\rangle = \cos\theta_2\,|00\rangle + \sin\theta_2\,|11\rangle$$

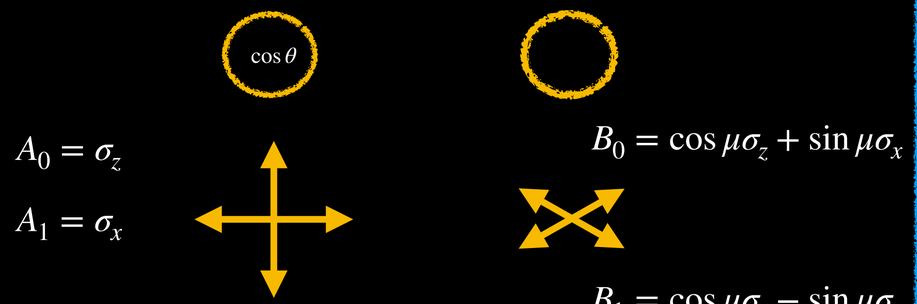
Bipartite states

Maximally entangled pair



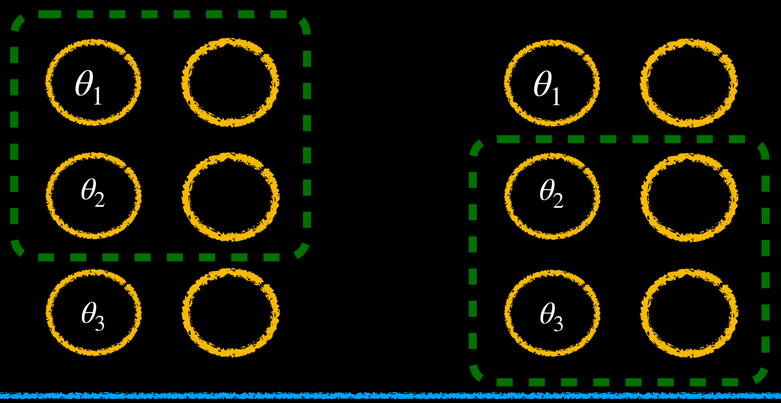
Summers, Werner 1987

Partially entangled pair



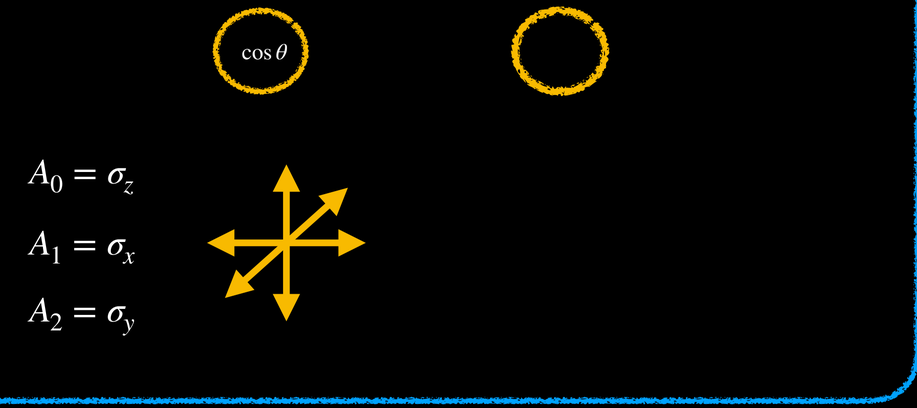
Bamps, Pironio 2015

Arbitrary bipartite pure state



Coladangelo, Goh, Scarani 2017

Full set of Paulis



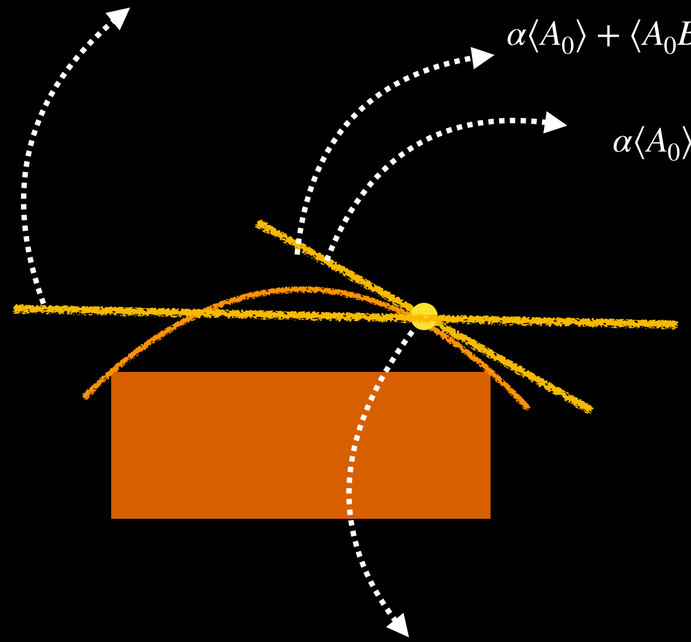
Woodhead, Kaniewski, Bourdoncle, Salavrakos, Bowles, Acin, Augusiak 2020

Bipartite states

$$\langle A_1 B_4 \rangle + \langle A_1 B_5 \rangle + \langle A_2 B_4 \rangle - \langle A_2 B_5 \rangle = 2\sqrt{2}\sin\theta$$

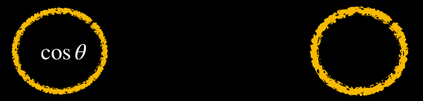
$$\alpha\langle A_0 \rangle + \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle = \sqrt{8+\alpha^2}$$

$$\alpha\langle A_0 \rangle + \langle A_0 B_2 \rangle + \langle A_0 B_3 \rangle + \langle A_2 B_2 \rangle - \langle A_1 B_3 \rangle = \sqrt{8+\alpha^2}$$



$$|\psi\rangle = \cos\theta\,|00\rangle + \sin\theta\,|11\rangle$$
$$A_0 = \sigma_z,\ A_1 = \sigma_x,\ A_2 = \sigma_y$$

Full set of Paulis



$A_0 = \sigma_z$

$A_1 = \sigma_x$

$A_2 = \sigma_y$



Woodhead, Kaniewski, Bourdoncle, Salavrakos, Bowles, Acin, Augusiak 2020

Multipartite states

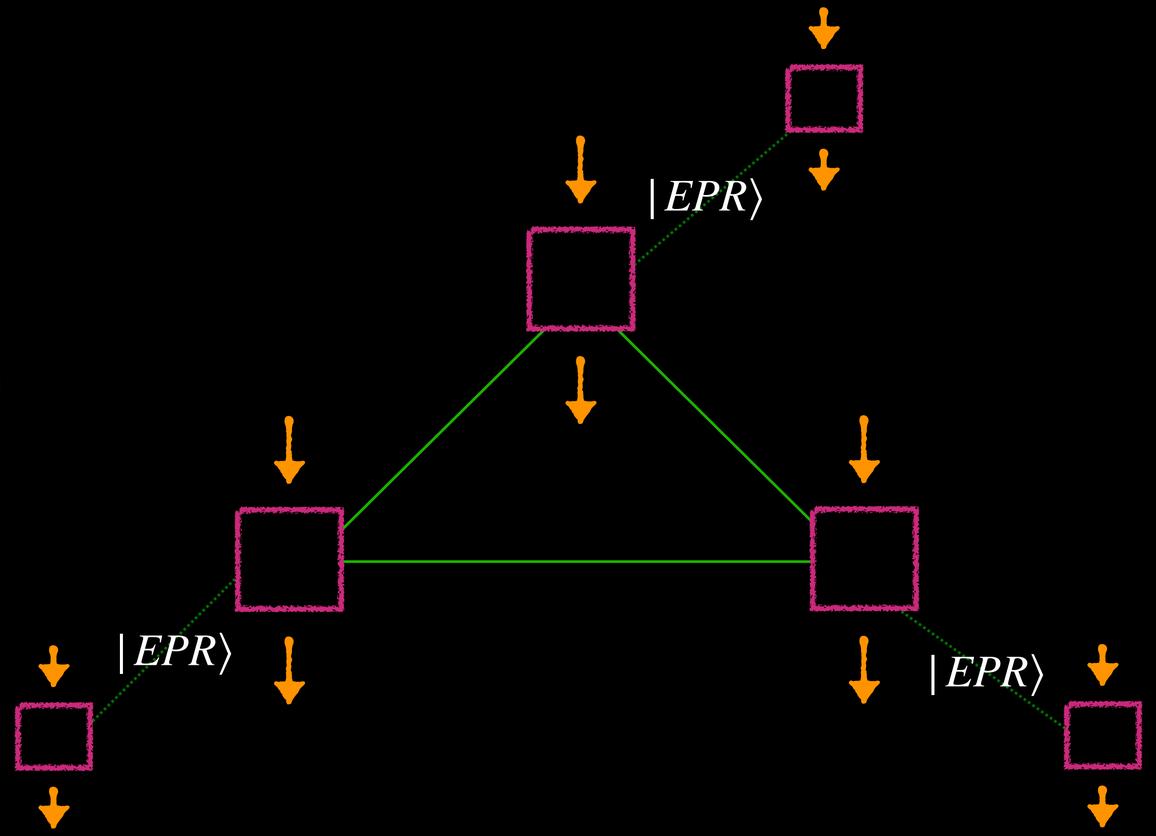
- Graph states — McKague 2011
- Dicke states — IŠ, Coladangelo, Augusiak, Acin 2018
- Schmidt states

Multipartite states

- Graph states
- Dicke states
- Schmidt states

McKague 2011

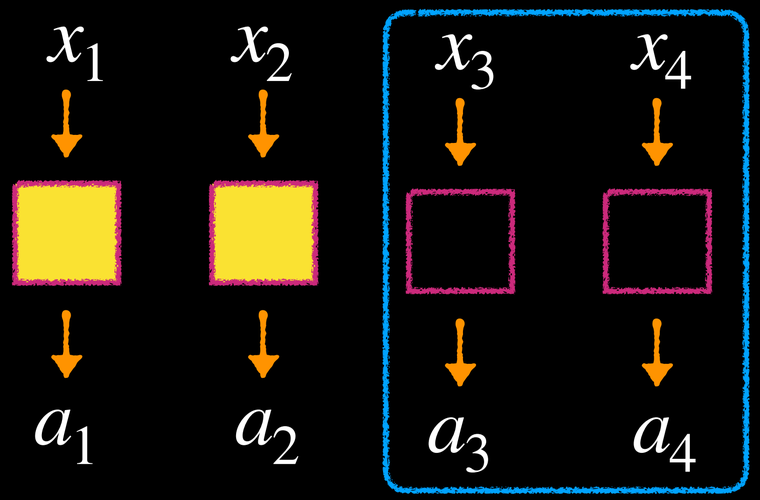
IŠ, Coladangelo, Augusiak, Acin 2018



IŠ, Bowles, Renou, Acin, Hoban 2023

Modular testing

Wu, Cai, Yang, Le, Bancal, Scarani 2014

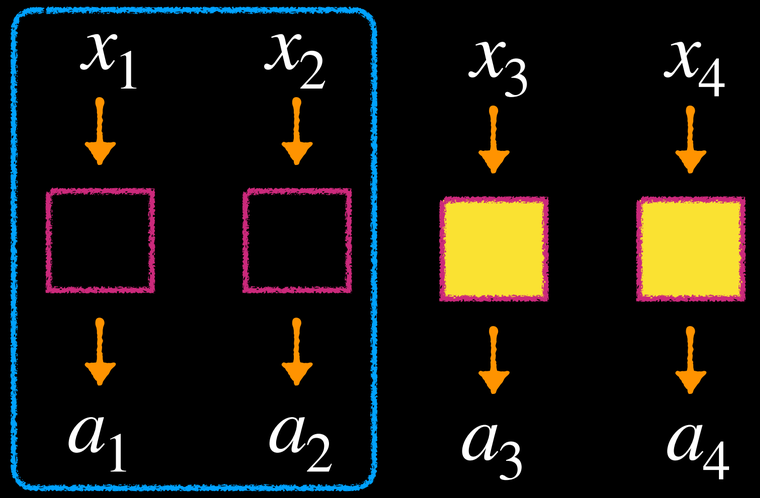


Projecting parties

Self-testing parties

Modular testing

Wu, Cai, Yang, Le, Bancal, Scarani 2014

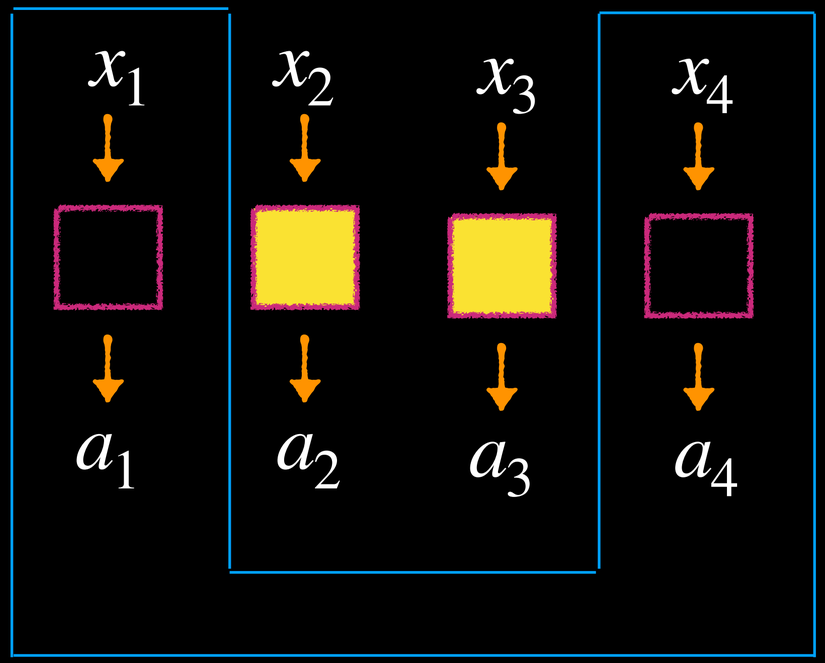


Projecting parties

Self-testing parties

Modular testing

Wu, Cai, Yang, Le, Bancal, Scarani 2014



Projecting parties

Self-testing parties

Tripartite primer

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle_A \otimes |\phi_+\rangle_{BC} + \frac{1}{\sqrt{2}}|1\rangle_A \otimes |\psi_-\rangle_{BC}$$

Obtain 0, self-test $|\phi_+\rangle$

Obtain 1, self-test $|\psi_-\rangle$

Tripartite primer

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle_A \otimes |\phi_+\rangle_{BC} + \frac{1}{\sqrt{2}}|1\rangle_A \otimes |\psi_-\rangle_{BC}$$

Obtain 0, self-test $|\phi_+\rangle$

Obtain 1, self-test $|\psi_-\rangle$

$$\varrho = \frac{1}{2}|0\rangle\langle 0| \otimes |\phi_+\rangle\langle\phi_+| + \frac{1}{2}|1\rangle\langle 1| \otimes |\psi_-\rangle\langle\psi_-|$$

Obtain 0, self-test $|\phi_+\rangle$

Obtain 1, self-test $|\psi_-\rangle$

Tripartite primer

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle_A \otimes |\phi_+\rangle_{BC} + \frac{1}{\sqrt{2}}|1\rangle_A \otimes |\psi_-\rangle_{BC}$$

Obtain 0, self-test $|\phi_+\rangle$

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Obtain 0, self-test $|\phi_+\rangle$

Obtain 1, self-test $|\psi_-\rangle$

Establish entanglement between A and BC !

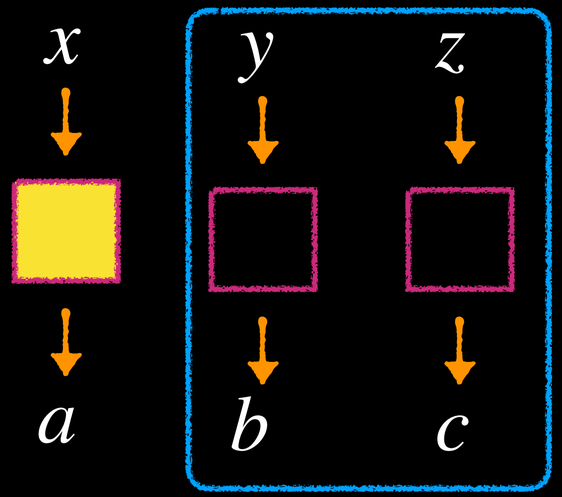
$$|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_-\rangle_{AB} \otimes |0\rangle_C + \frac{1}{\sqrt{2}}|\psi_+\rangle_{AB} \otimes |1\rangle_C$$

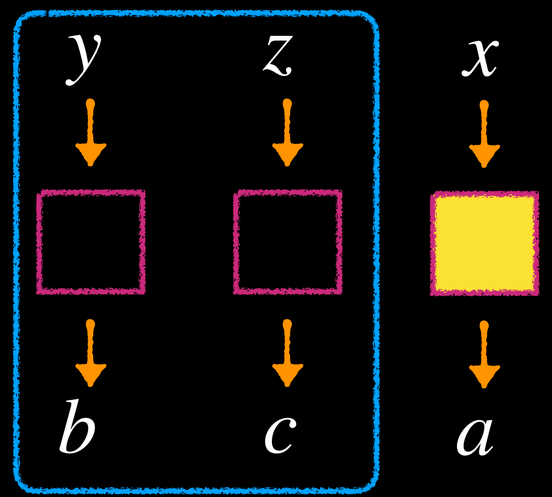
Obtain 0, self-test $|\phi_-\rangle$

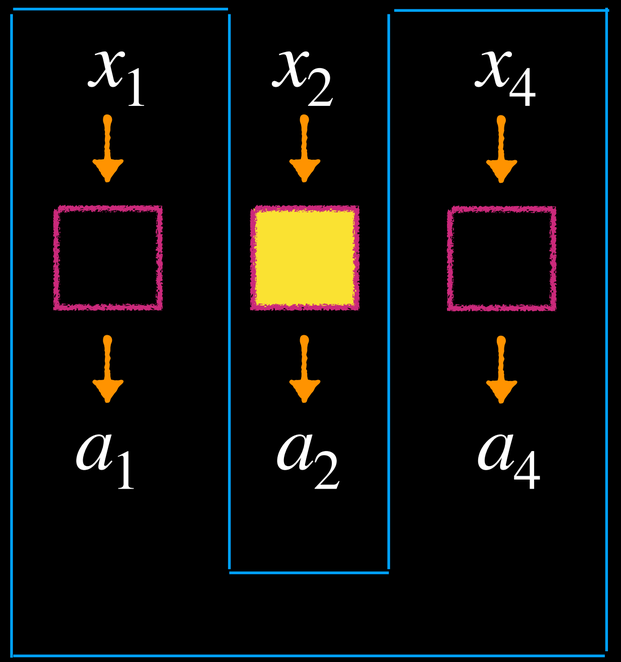
Obtain 0, self-test $|\psi_+\rangle$

Arbitrary tripartite

$$|\psi\rangle = \sum_{ijk} \lambda_{ijk} |i\rangle_A \otimes |j\rangle_B \otimes |k\rangle_C$$







$|0\rangle \quad \sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$

$|1\rangle \quad \sum_{jk} \lambda_{ijk} |j\rangle_B \otimes |k\rangle_C$

$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B \quad |0\rangle$

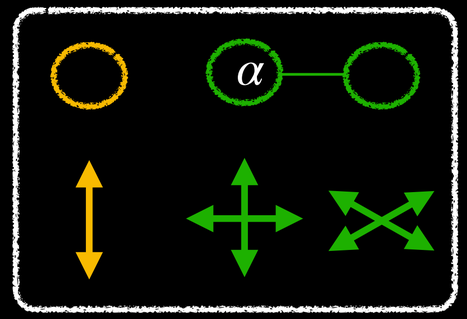
$\sum_{ij} \lambda_{ij1} |i\rangle_A \otimes |j\rangle_B \quad |1\rangle$

$|0\rangle \quad \sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$

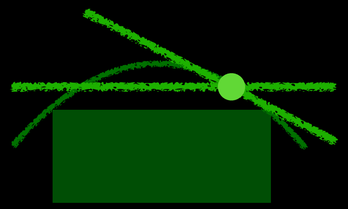
$|1\rangle \quad \sum_{ik} \lambda_{i1k} |i\rangle_A \otimes |k\rangle_C$

Hurdle

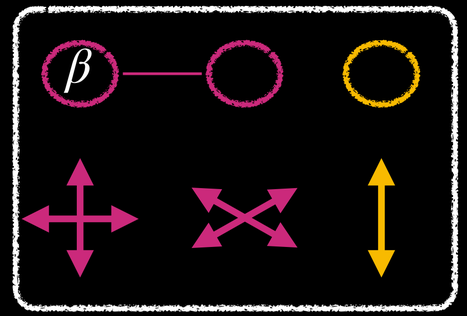
What we hope to get?



1st test



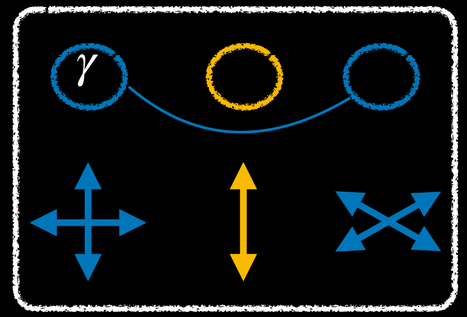
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$



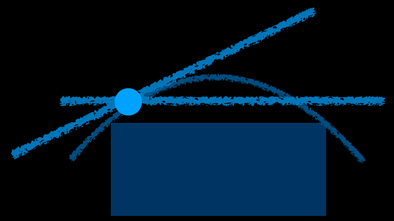
2nd test



$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$



3rd test



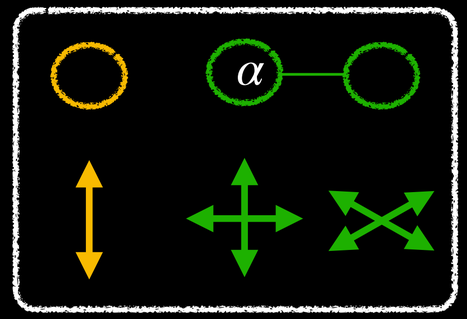
$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

Hurdle

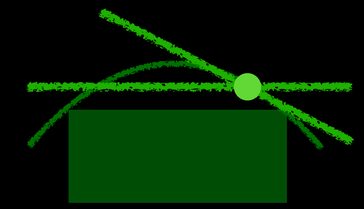
What we hope to get?

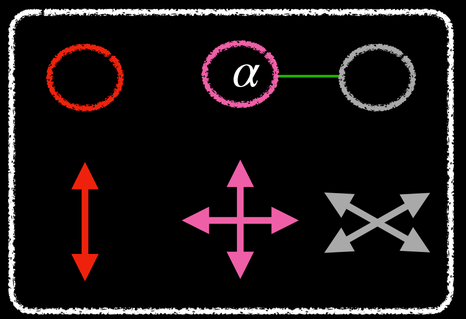
What we get?

1st test



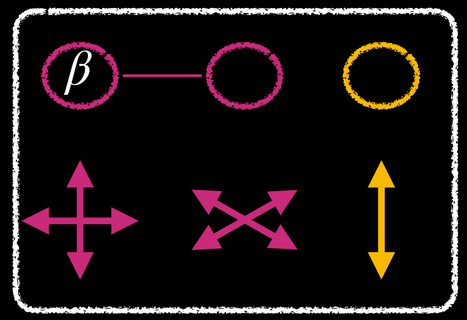
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$



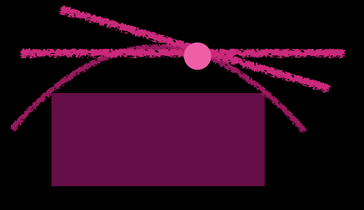


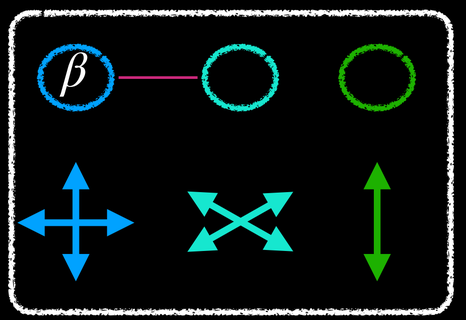
$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$

2nd test



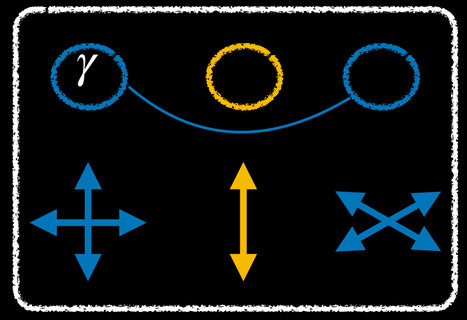
$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$



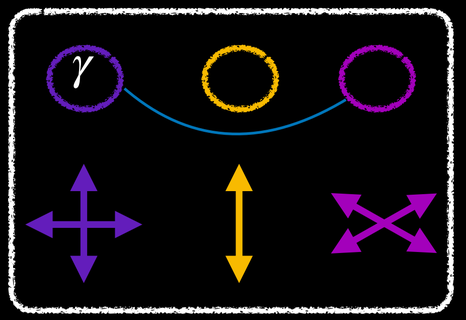


$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$

3rd test



$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

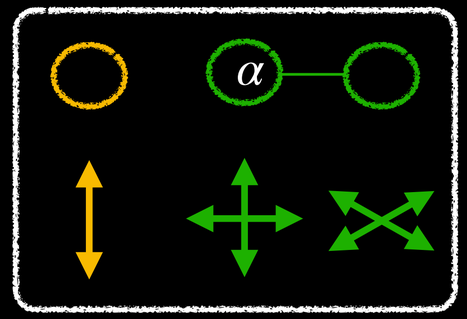


$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Hurdle

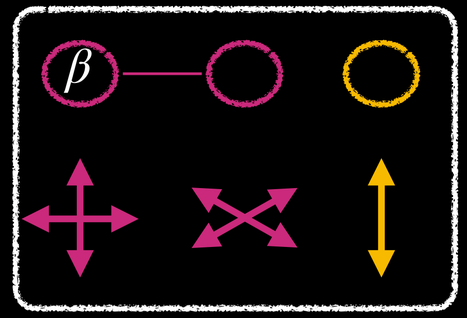
What we hope to get?

1st test



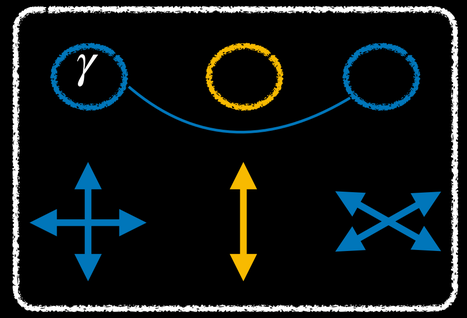
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$

2nd test

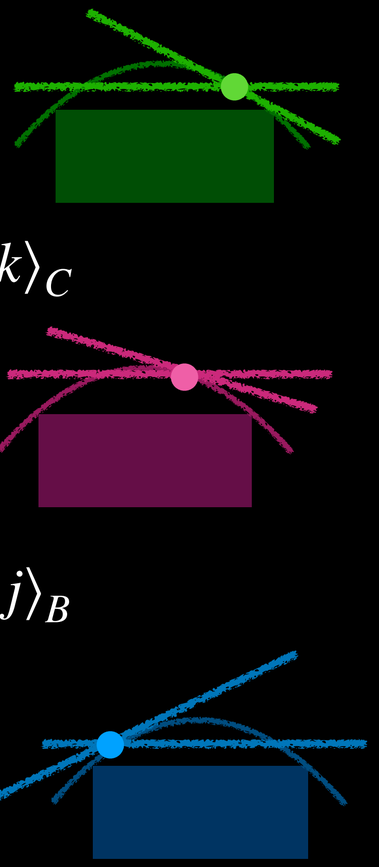


$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$

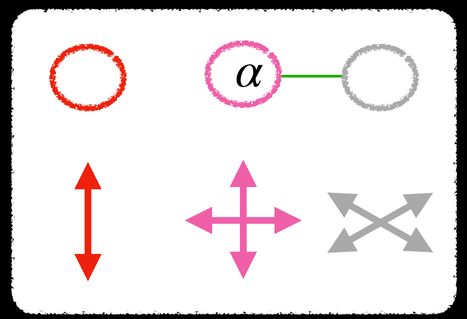
3rd test



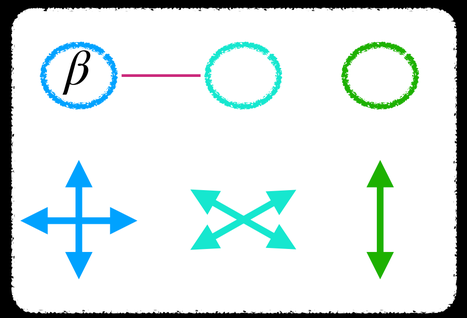
$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$



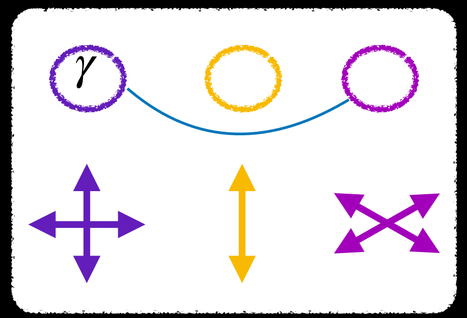
What we get?



$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$



$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$



$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Hurdle

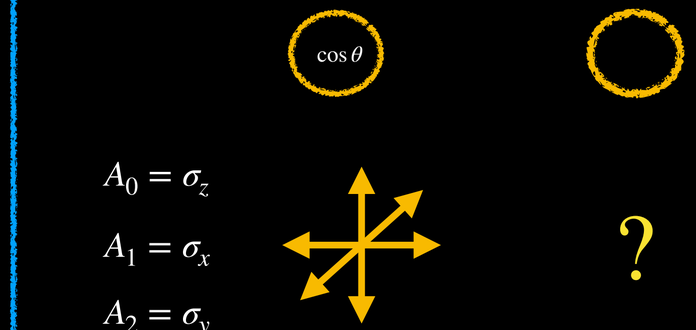
We need to fix the reference frame!

Solution

Measurement lemma

McKague 2013; Chen, Mančinska, Volčič 2024

Full set of Paulis

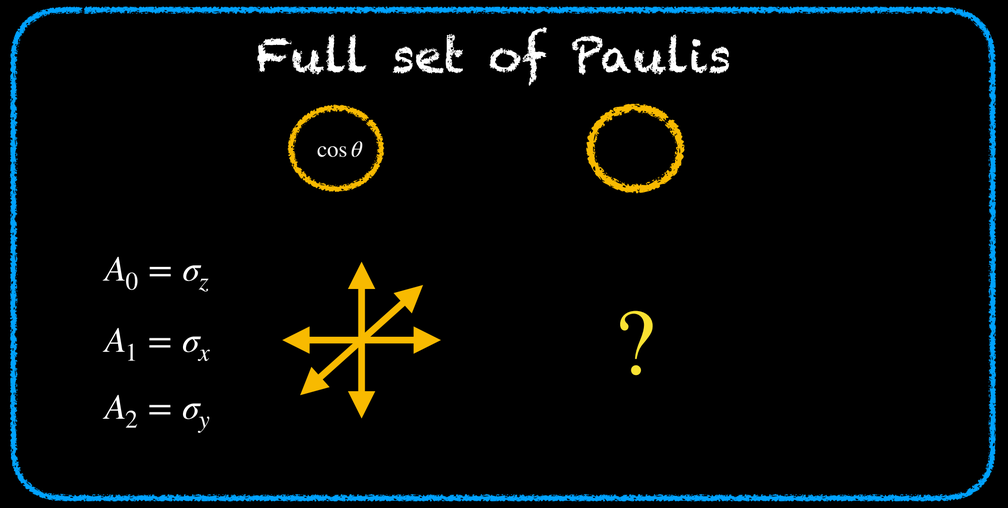


Correlations with tomographically complete set of measurements on a partially entangled state characterise the unknown measurement completely

Solution

Measurement lemma

McKague 2013; Chen, Mančinska, Volčič 2024



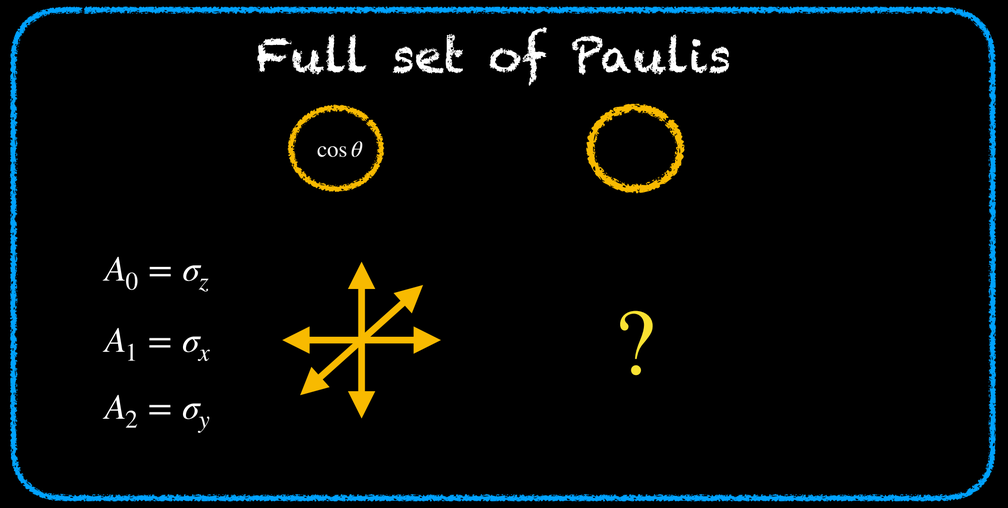
If $B = \alpha\sigma_z + \beta\sigma_x + \gamma\sigma_y$
the self-test will
reveal it as
$\alpha\sigma_z \otimes 1 + \beta\sigma_x \otimes 1 + \gamma\sigma_y \otimes T$

Correlations with tomographically complete set of measurements on a partially entangled state characterise the unknown measurement completely

Solution

Measurement lemma

McKague 2013; Chen, Mančinska, Volčič 2024



Correlations with tomographically complete set of measurements on a partially entangled state characterise the unknown measurement completely

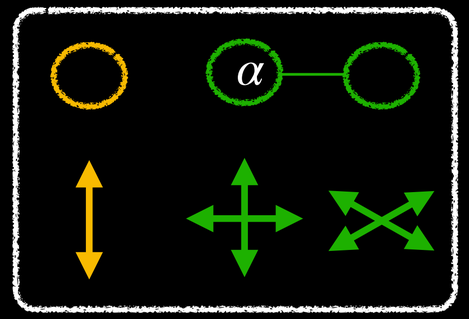
If $B = \alpha\sigma_z + \beta\sigma_x + \gamma\sigma_y$ the self-test will reveal it as $\alpha\sigma_z \otimes 1 + \beta\sigma_x \otimes 1 + \gamma\sigma_y \otimes T$

With T Hermitian we interpret it as measurement observable on a flag, indicating whether we have B or B^*

Solution?

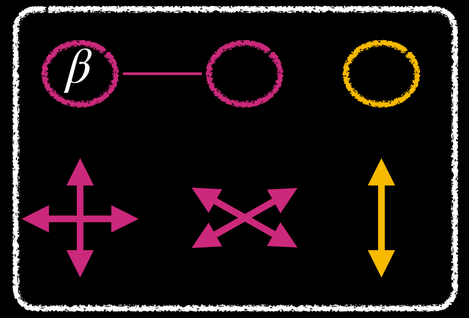
What we hope to get?

1st test



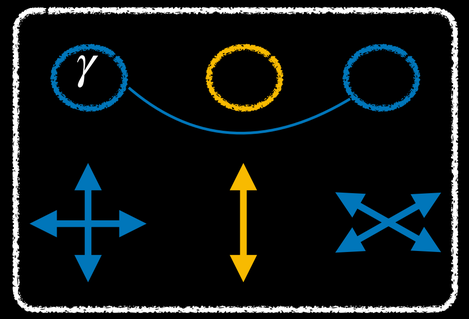
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$

2nd test



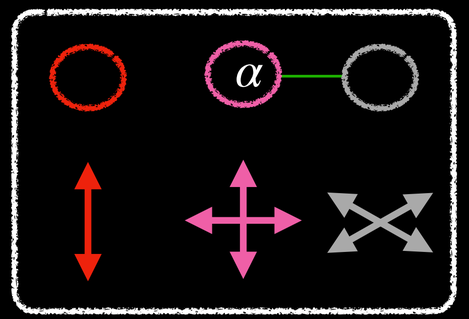
$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$

3rd test

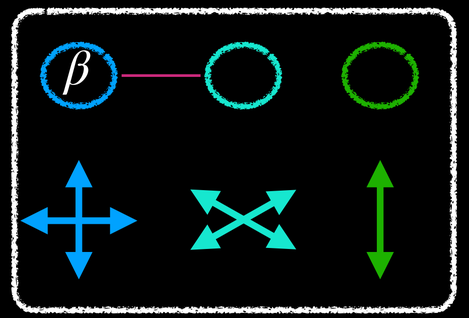


$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

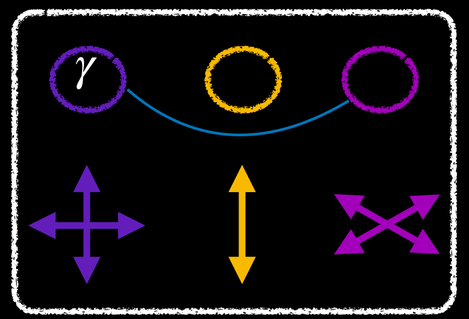
What we get?



$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$



$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$

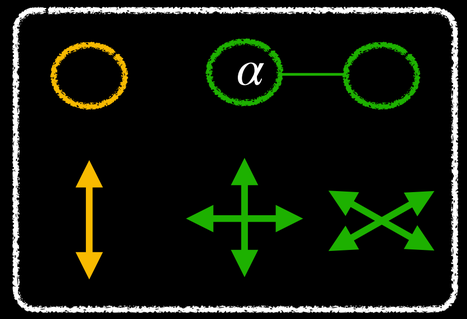


$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Solution

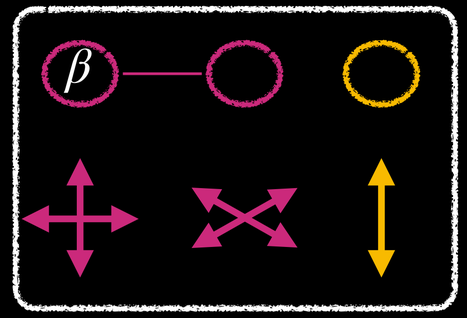
What we hope to get?

1st test



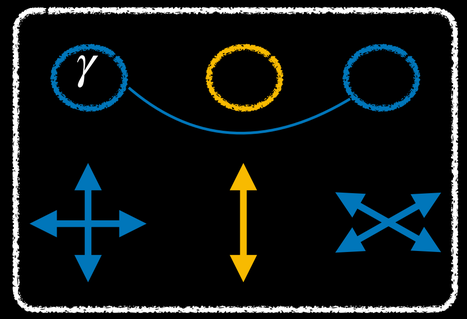
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$

2nd test



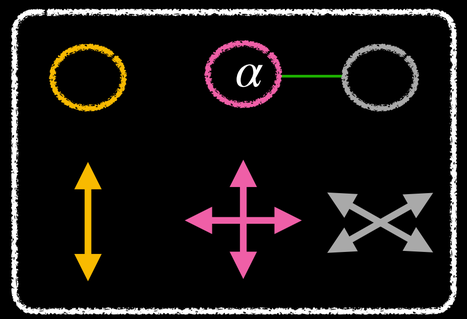
$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$

3rd test

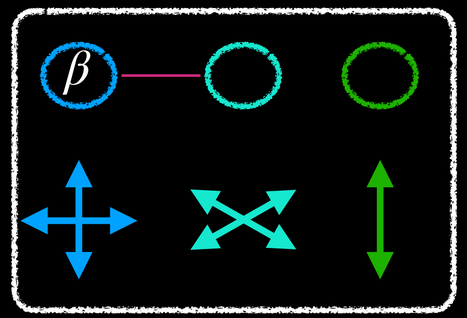


$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

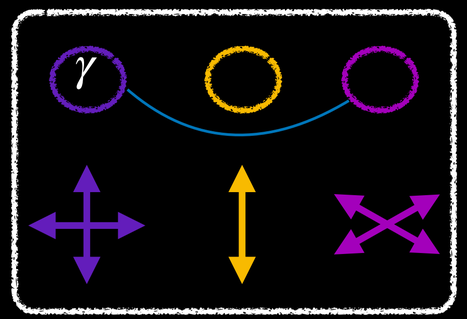
What we get?



$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$



$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$

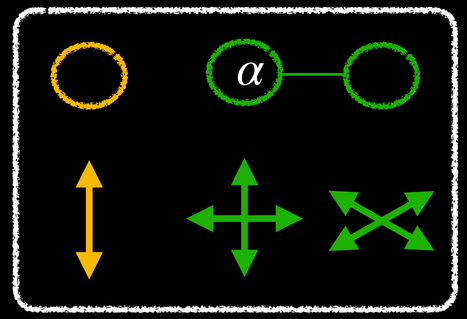


$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Solution

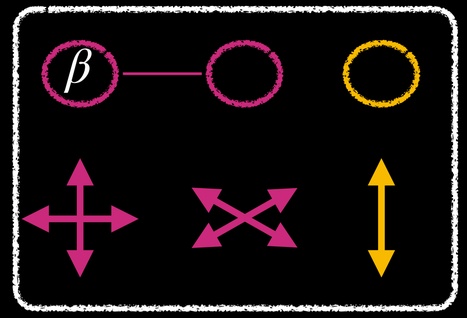
What we hope to get?

1st test



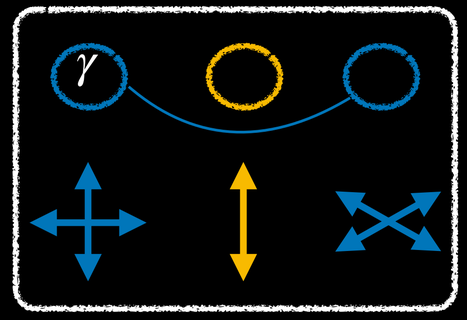
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$

2nd test



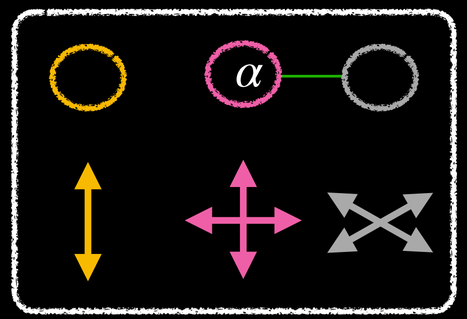
$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$

3rd test

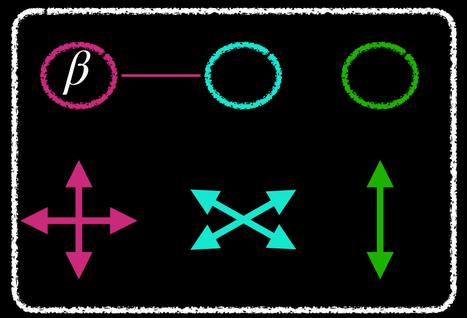


$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

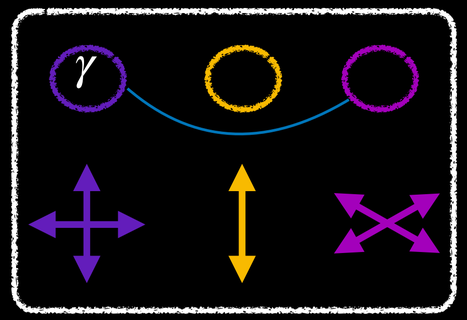
What we get?



$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$



$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$

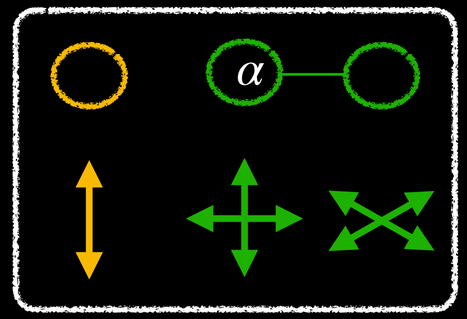


$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Solution

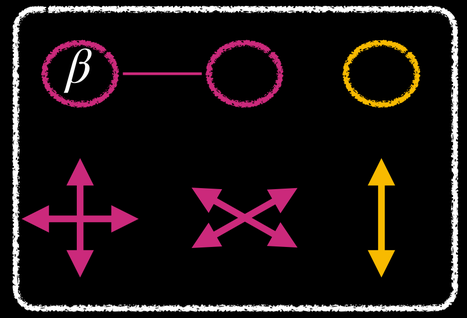
What we hope to get?

1st test



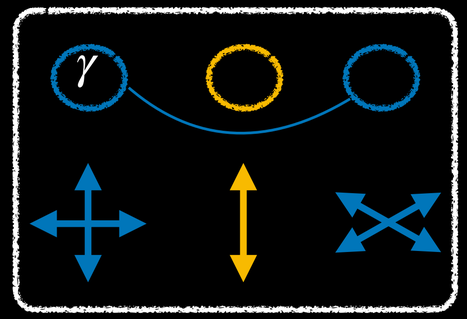
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$

2nd test



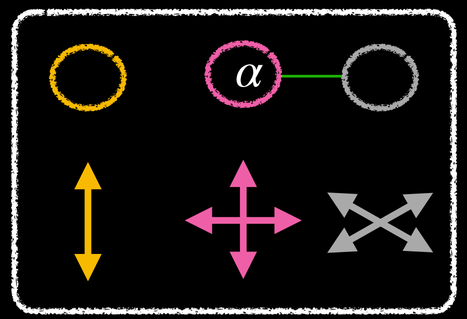
$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$

3rd test

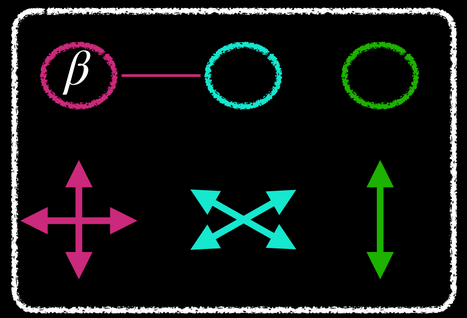


$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

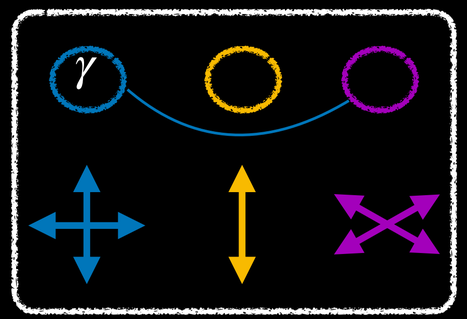
What we get?



$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$



$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$

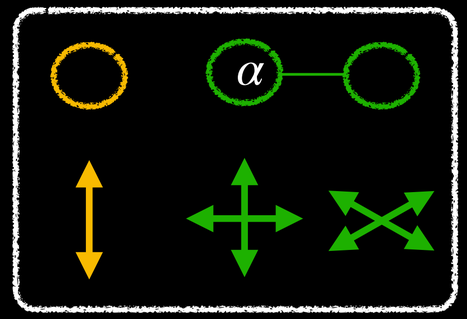


$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Solution

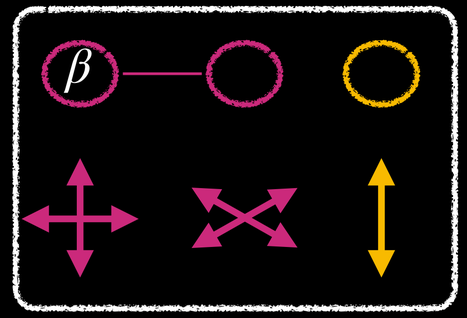
What we hope to get?

1st test



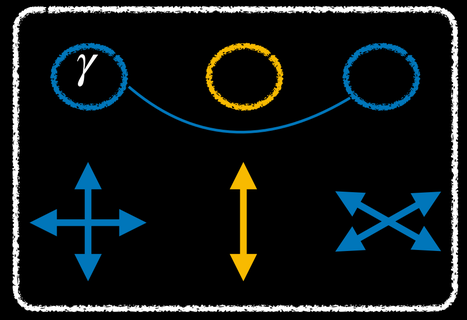
$$\sum_{jk} \lambda_{0jk} |j\rangle_B \otimes |k\rangle_C$$

2nd test



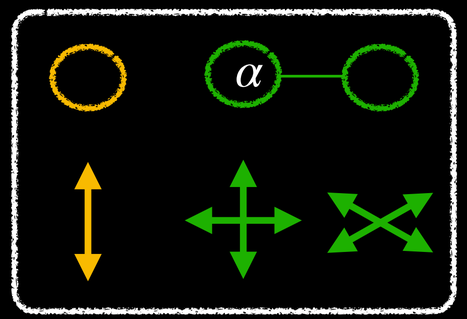
$$\sum_{ij} \lambda_{ij0} |i\rangle_A \otimes |j\rangle_B$$

3rd test

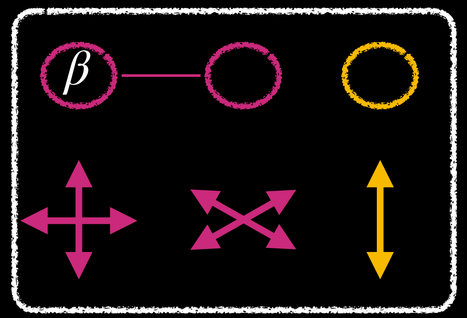


$$\sum_{ik} \lambda_{i0k} |i\rangle_A \otimes |k\rangle_C$$

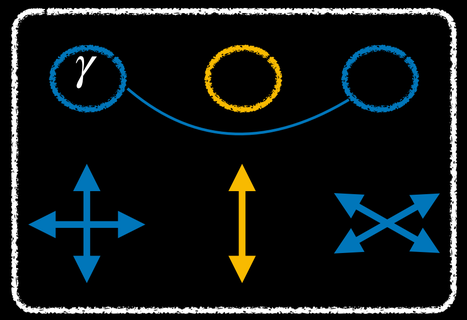
What we get?



$$\sum_{jk} \lambda_{0jk} |\tilde{j}\rangle_B \otimes |\tilde{k}\rangle_C$$

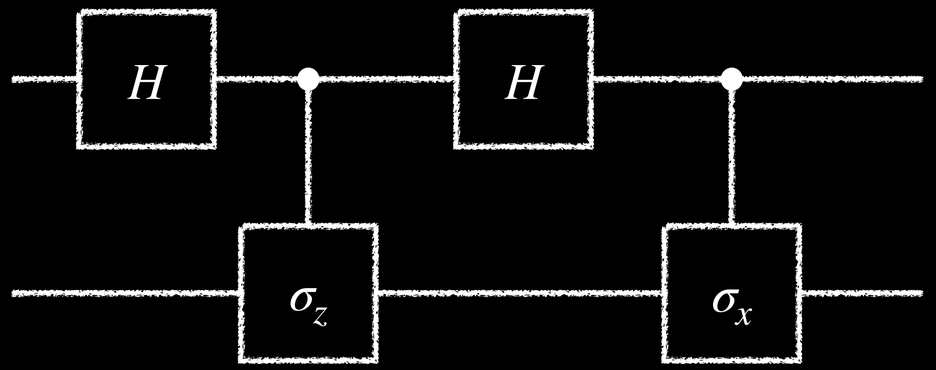


$$\sum_{ij} \lambda_{ij0} |\tilde{i}\rangle_A \otimes |\bar{j}\rangle_B$$



$$\sum_{ik} \lambda_{i0k} |\bar{i}\rangle_A \otimes |\bar{k}\rangle_C$$

Isometry



$$\Phi_A \otimes \Phi_B \otimes \Phi_C(|\psi'\rangle \otimes |000\rangle) = p\,|\xi_0\rangle \otimes |\psi\rangle + \sqrt{1-p^2}\,|\xi_1\rangle \otimes |\psi^*\rangle$$

Open questions

- Robustness works in a pedestrian way: is there a general theorem ensuring robustness?
- Is there a state which cannot be robustly self-tested?
- Can we make number of measurements constant?
- What happened with qudits?